Вычисление производной по правилу произведения

$$\left(U \cdot V\right)' = U' \cdot V + U \cdot V'$$

1) $y = x \cdot \sin x$;

$$y' = (\underbrace{x}_{U} \cdot \underbrace{\sin x}_{V})' = (x)' \cdot \sin x + x \cdot (\sin x)' =$$

$$= 1 \cdot \sin x + x \cdot \cos x = \sin x + x \cdot \cos x$$

$$(U\cdot V)' = U'\cdot V + U\cdot V'$$

2) $y = x\cdot e^x$

$$y' = \left(x\cdot e^x\right)' = x'\cdot e^x + x\cdot\left(e^x\right)' =$$

$$= 1\cdot e^x + x\cdot e^x = e^x\cdot(1+x)$$

3) $y = \left(x^2-3x\right)\cdot\left(2x^3-8\right)$

$$y' = \left(x^2-3x\right)'\cdot\left(2x^3-8\right)+\left(2x^3-8\right)'\cdot\left(x^2-3x\right)=$$

$$=(2x-3)\cdot\left(2x^3-8\right)+\left(6x^2-0\right)\cdot\left(x^2-3x\right)=$$

$$=4x^4-16x-6x^3+24+6x^4-18x^3=$$

$$=10x^4-24x^3-16x+24$$

$$\left(U\cdot V\right)' = U'\cdot V + U\cdot V'$$

4) $y'(4) - y'(1) = ?$

$$y = (1-2x)\cdot\sqrt{x}$$

$$y' = (1-2x)'\cdot\sqrt{x} + \left(\sqrt{x}\right)'\cdot(1-2x) =$$

$$= (0-2)\cdot\sqrt{x} + \left(\frac{1}{2\sqrt{x}}\right)\cdot(1-2x) = -2\sqrt{x} + \frac{1-2x}{2\sqrt{x}}$$

$$y'(4) = -2\sqrt{4} + \frac{1-2\cdot 4}{2\sqrt{4}} = -4 + \frac{-7}{4} = -5{,}75$$

$$y'(1) = -2\sqrt{1} + \frac{1-2\cdot 1}{2\sqrt{1}} = -2 + \frac{-1}{2} = -2{,}5$$

$$y'(4) - y'(1) = -5{,}75 - (-2{,}5) = -3{,}25$$

Вычисление производной по правилу дроби

$$\left(\frac{U}{V}\right)' = \frac{U'V - UV'}{V^2}$$

1) $y = \dfrac{1+2x}{3-5x};$

$$y' = \left(\frac{1+2x}{3-5x}\right)' = \frac{(1+2x)'(3-5x) - (1+2x)(3-5x)'}{(3-5x)^2} =$$

$$= \frac{2(3-5x) - (1+2x)(-5)}{(3-5x)^2} = \frac{6-10x+5+10x}{(3-5x)^2} = \frac{11}{(3-5x)^2}$$

$$\left(\frac{U}{V}\right)' = \frac{U'V - UV'}{V^2}$$

2) $$y = \frac{1+\sin x}{1-\sin x};$$

$$y' = \left(\frac{1+\sin x}{1-\sin x}\right)' = \frac{(1+\sin x)'(1-\sin x) - (1+\sin x)(1-\sin x)'}{(1-\sin x)^2} =$$

$$= \frac{\cos x \cdot (1-\sin x) - (1+\sin x)\cdot(-\cos x)}{(1-\sin x)^2} =$$

$$= \frac{\cos x - \cos x \cdot \sin x + \cos x + \sin x \cdot \cos x}{(1-\sin x)^2} = \frac{2\cos x}{(1-\sin x)^2}$$

Вычислить производную в заданной точке:

3) $$y = \frac{2-3x^2}{1+2x} \qquad x_0 = -1$$

$$y' = \left(\frac{2-3x^2}{1+2x}\right)' = \frac{\left(2-3x^2\right)'(1+2x)-\left(2-3x^2\right)(1+2x)'}{(1+2x)^2} =$$

$$= \frac{(-6x)\cdot(1+2x)-\left(2-3x^2\right)\cdot(2)}{(1+2x)^2} = \frac{-6x-12x^2-4+6x^2}{(1+2x)^2} =$$

$$= \frac{-6x-6x^2-4}{(1+2x)^2} \qquad y'(-1) = \frac{6-6-4}{(1-2)^2} = -4$$

4) $y = \dfrac{6x - 3\ln x}{2 + x^2}$;

$$\left(\frac{U}{V}\right)' = \frac{U'V - UV'}{V^2}$$

$$y' = \left(\frac{6x - 3\ln x}{2 + x^2}\right)' = \frac{(6x - 3\ln x)'(2 + x^2) - (6x - 3\ln x)(2 + x^2)'}{(2 + x^2)^2} =$$

$$= \frac{\left(6 - \frac{3}{x}\right)\cdot(2 + x^2) - (6x - 3\ln x)\cdot(2x)}{(2 + x^2)^2} =$$

$$= \frac{12 + 6x^2 - \frac{6}{x} - 3x - 12x^2 + 6x\ln x}{(2 + x^2)^2} = \frac{6x\ln x - \frac{6}{x} - 6x^2 - 3x + 12}{(2 + x^2)^2}$$

Самостоятельно вычислить производную в заданной точке:

1) $y = (3 - x^2) \cdot \sqrt{x}$ $\quad y'(25) - y'(4) - ?$

2) $y = (4x - x^2) \cdot (x - 2e^x)$ $\quad y'(0) - ?$

3) $y = (2x^3 + 8e^x) \cdot \cos x$ $\quad y'(0) - ?$

4) $y = \dfrac{2 - 4x^3}{2 - x}$ $\quad y'(-6) - ?$

5) $y = \dfrac{9 \ln x + 4x}{2x^2}$ $\quad y'(1) - ?$